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**Equilibrium simulation with microeconomic models.**  
***A new procedure with an application to income support policies***

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**Abstract**

Many microeconomic models of discrete labour supply include alternative-specific constants meant to account for (possibly besides other factors) the density or accessibility of particular types of jobs (e.g. part-time jobs vs. full-time jobs). The most common use of these models is the simulation of tax-transfer reforms. The simulation is usually interpreted as a comparative statics exercise, i.e. the comparison of different *equilibria* induced by different policy regimes. The simulation procedure, however, typically keeps fixed the estimated alternative-specific constants. In this note we argue that this procedure is not consistent with the comparative statics interpretation. Since the constants reflect the number of jobs and since the number of people willing to work changes as a response to the change in tax-transfer regime, the new equilibrium induced by the reform implies that the constants should also change. A structural interpretation of the alternative-specific constants leads to the development of a simulation procedure consistent with the comparative statics interpretation. The procedure is illustrated with a simulation of alternative reforms of the income support policies in Italy.

JEL Classification: C35, C53, H31, J22.

Keywords: Random Utility, Discrete Choice, Labour Supply, Policy Simulation, Alternative-Specific Constants, Equilibrium Simulation.

## 1. Introduction

A common practice in the specification of models of labour supply based on the discrete choice approach consists of introducing alternative-specific constants, which should account for a number of factors such as the different density or accessibility of different types of jobs, search or fixed costs and systematic utility components otherwise not accounted for. In the basic framework, the household chooses among  $H+1$  alternatives or “job” types  $j = 0, 1, \dots, H$ , with  $j = 0$  denoting non-participation (a “non-market job”). Let  $V(i, j; w_i, T) + \varepsilon_{ij}$  denote the utility attained by household  $i$  if a job of type  $j$  is chosen, given wage rate  $w_i$  and tax-transfer regime  $T$ , where  $V(i, j; w_i, T)$  is the systematic part (containing observed variables) of the utility function and  $\varepsilon_{ij}$  is a random component. Depending on the application and the available data, the job types might be defined in terms of one or many of the following attributes: weekly hours of work, sector of employment, occupational level type of contract etc. By assuming that  $\varepsilon_{ij}$  is i.i.d. Type I extreme value, we get the familiar Multinomial Logit expression for the probability that a job of type  $j$  is chosen by household  $i$ :

$$P(i, j; w, T) = \frac{\exp\{V(i, j; w_i, T)\}}{\sum_k \exp\{V(i, k; w_i, T)\}} \quad (1)$$

Model (1) usually does not fit the data very well. For example Van Soest (1995) notes that the model over-predicts the number of people working part-time. More generally, certain types of jobs might differ according to a number of systematic factors that are not accounted for by the observed variables contained in  $V$ : (a) availability or density of job-types; (b) fixed costs; (c) search costs; (d) systematic utility components. What might be called the “dummies refinement” is a simple way to account for those factors. Let us define subsets  $\{S_\ell\}$  of the set of job types  $0, 1, \dots, H$  and the corresponding indicator functions  $\{D(j \in S_\ell)\}$  such that

$D(e) = 1$  if and only if  $e$  is true. Clearly the definition of the subsets should reflect some hypothesis upon the differences among the job types with respect to the factors (a) – (b) mentioned above. Now we specify the choice probability as follows

$$P(i, j; w_i, T) = \frac{\exp \left\{ V(i, j; w_i, T) + \sum_{\ell} \mu_{\ell} D(j \in S_{\ell}) \right\}}{\sum_{k=0}^H \exp \left\{ V(i, k; w_i, T) + \sum_{\ell} \mu_{\ell} D(k \in S_{\ell}) \right\}} \quad (2)$$

Many papers – although with differing focus and motivation – have adopted a similar procedure, e.g.: Van Soest (1995), Aaberge et al. (1995, 1999), Kalb (2000), Dagsvik et al. (2006), Kornstad et al. (2007) and Colombino et al. (2010); see also the survey by Creedy and Kalb (2005).

Expression (2) can be interpreted as embodying the assumption that certain jobs, beyond the contributions attributable to the observed characteristics, bring a systematic additive utility contribution, due to a number of unobserved systematic factors including their accessibility. More generally, the systematic unobserved contributions could be entered in a non-additive forms or could be measured in terms of income rather than utility. For example, another common procedure consists of subtracting from the income term (in the utility function) a parameter (usually called “participation cost” or “fixed cost of working”) whenever the job is a “market job”. In what follows, we will refer to the formulation of expression (2); however the analysis we propose is equally relevant for other formulation such as the fixed-cost approach.

The main use of microeconomic models of labour supply consists of the simulation of tax-transfer reforms. The standard simulation proceeds as follows. Once  $V(\cdot)$  and the  $\{\mu_{\ell}\}$  are estimated, the current tax regime  $T$  is replaced by a “reform”  $R$  and a new distribution of choices is simulated using expression (2). All the authors adopting the “dummies refinement”

so far have performed the simulations by leaving the  $\{\mu_\ell\}$  unchanged. The policy simulation is most commonly interpreted as a comparative statics exercise, where different *equilibria* – induced by different tax-transfer regimes – are compared. In this note we claim that the standard procedure in general is not consistent with the comparative statics interpretation. According to a basic notion of equilibrium, the number of people willing to work must be equal to the number of available jobs. Since the  $\{\mu_\ell\}$  reflect – at least in part, depending on the interpretations – the number and the composition of available jobs, and since the number of people willing to work and their distribution across different job types in general change as a consequence of the reforms, it follows that in general the  $\{\mu_\ell\}$  must also change. Building on a matching model developed by Dagsvik (1994, 2000), the basic random utility approach can be extended to include random choice sets and provide a structural interpretation of the “dummies refinement” that leads very naturally to a simulation procedure consistent with comparative statics.<sup>1</sup>

The procedure is explained in Sections 2, 3 and 4. Section 5 illustrates an empirical example. Section 6 contains the conclusions.

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<sup>1</sup> A different procedure for equilibrium simulation – which however would not be appropriate for the class of microeconomic models considered here – has been proposed by Creedy and Duncan (2005).

## 2. A structural interpretation of the “dummies refinement”

We consider here a single individual. The generalization to couples is developed in Section 4. Building on Dagsvik (1994), a series of papers (among others: Aaberge et al. 1995, Aaberge et al. 1999, Dagsvik and Strøm 2006, Aaberge and Colombino 2012a and 2012b) adopt an approach where there are “many” jobs that belong to each type  $j$  and a particular job  $z$  of type  $j$  produces a utility level  $V(i, j; w_i, T) + \varepsilon_{ij}(z)$ , so that  $V(i, j; w_i, T) + \varepsilon_{ij}$  is to be interpreted as follows:

$$V(i, j; w_i, T) + \varepsilon_{ij} = \max_z V(i, j; w_i, T) + \varepsilon_{ij}(z). \quad (3)$$

We let  $g_j$  denote the number of available jobs of type  $j$ . The term  $g_j$  can be interpreted as reflecting the demand side. In general it might be both job-specific and individual-specific but for simplicity of exposition we treat it here as common to all individuals. By assuming that  $\varepsilon_{ij}$  is i.i.d. Type I extreme value, the probability that individual  $i$  is matched to a job of type  $j$  turns out to be:

$$P(i, j; w_i, T) = \frac{\exp\{V(i, j; w_i, T)\} g_j}{\sum_{k=0}^H \exp\{V(i, k; w_i, T)\} g_k} \quad (4)$$

Dagsvik (2000) shows that expression (4) can be derived as a special case of a model where the agents (firms and workers) play a game leading to stable matching equilibrium (e.g. the deferred acceptance game).

By defining  $J = \sum_{k=1}^M g_k$  = total number of available market jobs,  $J_0 = J/g_0$  and  $\tilde{g}_j = g_j/J$ ,

expression (4) can be rewritten as follows:

$$P(i, j; w_i, T) = \begin{cases} \frac{\exp\{V(i, j; w_i, T)\} J_0 \tilde{g}_j}{\exp\{V(i, 0; w_i, T)\} + \sum_{k=1}^H \exp\{V(i, k; w_i, T)\} J_0 \tilde{g}_k} & \text{if } j > 0 \\ \frac{\exp\{V(i, 0; w_i, T)\}}{\exp\{V(i, 0; w_i, T)\} + \sum_{k=1}^H \exp\{V(i, k; w_i, T)\} J_0 \tilde{g}_k} & \text{if } j = 0 \end{cases}. \quad (5)$$

If we specify

$$\begin{aligned} J_0 &= \exp(\delta_0) \\ \tilde{g}_k &= \gamma \exp(\mu_k) \end{aligned} \quad (6)$$

we get a “dummy refinement” representation of the choice probability:

$$P(i, j; w_i, T) = \frac{\exp\left\{V(i, j; w_i, T) + \mu_0 D(j > 0) + \sum_{t=2}^H \mu_t D(t = j)\right\}}{\sum_{k=0}^H \exp\left\{V(i, k; w_i, T) + \mu_0 D(k > 0) + \sum_{t=2}^H \mu_t D(t = k)\right\}} \quad (7)$$

where  $\mu_0 \equiv \ln \gamma + \delta_0$ . Notice that we drop  $D(t = 1)$  since we set  $t = 1$  as a reference type.

Expression (6) specifies a very general form of the conditional densities  $\tilde{g}_1, \tilde{g}_2, \dots, \tilde{g}_M$ . In

empirical applications we are usually interested in much more specific forms, for example a uniform distribution with “peaks”:

$$\tilde{g}_k = \begin{cases} \gamma \exp(\mu_\ell) & \text{if } k \in S_\ell, \ell = 1, \dots, L \\ \gamma & \text{otherwise} \end{cases} \quad (8)$$

where  $S_1, \dots, S_L$  are  $L$  disjoint subsets of the job-type indexes  $1, 2, \dots, H$ . In this case we end up with:

$$P(i, j; w_i, T) = \frac{\exp\left\{V(i, j; w_i, T) + \mu_0 D(j > 0) + \sum_{\ell=2}^L \mu_\ell D(j \in S_\ell)\right\}}{\sum_{k=0}^H \exp\left\{V(i, k; w_i, T) + \mu_0 D(k > 0) + \sum_{\ell=2}^L \mu_\ell D(k \in S_\ell)\right\}}. \quad (9)$$

The dummies’ coefficients have therefore the following interpretation:

$$\mu_0 = \ln \left( \frac{J}{\gamma g_0} \right) \quad (10)$$

$$\mu_\ell = \ln \left( \frac{J_\ell / J}{n(S_\ell) \gamma} \right) \quad (11)$$

where

$n(S_\ell)$  = number of type in  $S_\ell$ ,  $\ell = 2, \dots, L$ .

The presence of factors other than jobs density (e.g. unobserved systematic costs or benefits specific of different job types) is not incompatible with expressions (10) and (11): more generally  $\gamma g_0$  and  $n(S_\ell) \gamma$  might be interpreted as normalizing constants that include the effect of those other factors. Note that  $g_0$  and  $\gamma$  can be retrieved using expressions (10) - (11) and the observed values of  $J$ ,  $J_\ell$  and  $n(S_\ell)$ ,  $\ell = 1, \dots, L$ .

### 3. Equilibrium conditions

For simplicity of exposition in this section we assume that  $\tilde{g}_k = \gamma$ ,  $k = 1, \dots, H$ , so that the model contains only the dummy  $D$ :

$$P(i, j; w_i, T) = \frac{\exp\{V(i, j; w_i, T) + \mu_0 D(j > 0)\}}{\sum_{k=0}^H \exp\{V(i, k; w_i, T) + \mu_0 D(k > 0)\}}. \quad (12)$$

Let us assume that the number of available jobs  $J$  depends on the moments  $\vartheta$  of the wage distribution. In what follows we will refer interchangeably to  $\vartheta$  as to the moments or to the distribution defined by those moments:

$$J = J(\vartheta). \quad (13)$$

For simplicity we assume here and in the empirical exercise of Section 5 that that equilibrium wage distribution is such that the number of market jobs and the number of people willing to work are equal, while the number worked hours accommodate the households' preferences.



The framework can be easily extended to the case where both the number of jobs and the number of hours are allocated through the mechanism of equilibrium wages. With  $w_i(\vartheta_R)$  we will denote the wage rate of individual  $i$  in the equilibrium wage distribution induced by tax-transfer regime  $R$ .

It is important to distinguish the case of elastic labour demand from the limit cases of perfectly inelastic and perfectly elastic labour demand.

### ***Elastic demand***

Using (10) and (13) we can write:

$$\mu_0 = \mu_0(\vartheta) \quad (14)$$

We then define  $\pi_i(T, \vartheta_T, \mu_0(\vartheta_T))$  as the probability that individual  $i$  is working given the tax-transfer regime  $T$  and the wage distribution  $\vartheta$ :

$$\pi_i(T, \vartheta_T, \mu_0(\vartheta_T)) \equiv \frac{\sum_{j=1}^H \exp\{V(i, j; w_i(\vartheta_T), T) + \mu_0(\vartheta_T)D(j > 0)\}}{\sum_{k=0}^M \exp\{V(i, k; w_i(\vartheta_T), T) + \mu_0(\vartheta_T)D(k > 0)\}} \quad (15)$$

where  $w_i(\vartheta_T)$  is the wage rate of individual  $i$  given the wage distribution  $\vartheta_T$ . Assuming that the observed (or simulated) choices under the current tax-transfer regime  $T$  correspond to an equilibrium, we must have:

$$\sum_i \pi_i(T, \vartheta_T, \mu_0(\vartheta_T)) = J(\vartheta_T). \quad (16)$$

In a comparative statics perspective, an analogous condition must hold under the “reform”  $R$ :

$$\sum_i \pi_i(R, \vartheta_R, \mu_0(\vartheta_R)) = J(\vartheta_R) \quad (17)$$

where  $\vartheta_R$  denotes the new equilibrium wage distribution.

### ***Perfectly elastic demand***

When the demand for labour is perfectly elastic, the market is always in equilibrium at the initial wage rate. However, since the number of working people in general will change under a new tax-transfer rule and since the number of jobs in equilibrium must be equal to the number of people willing to work, it follows that the parameter  $\mu_0 = \ln\left(\frac{J}{\gamma g_0}\right)$  must change.

Let us rewrite expression (10) as  $J = \gamma g_0 e^{\mu_0}$ . Then the equilibrium condition can be written as follows:

$$\sum_i \pi_i(R, v_T, \mu_{0R}) = \gamma g_0 e^{\mu_{0R}}. \quad (18)$$

In this case the distribution  $v$  remains fixed. Instead  $\mu_{0R}$  must be directly adjusted so as to fulfil condition (18). The case with fixed wage distribution and demand absorbing any change in supply actually corresponds to the scenario implicitly assumed in most tax-transfer simulations: however those simulations do not take condition (18) into account.

### ***Perfectly inelastic demand***

In the special case of a perfectly inelastic demand (zero elasticity), the number of jobs remains fixed but the wage rate must be adjusted so that the number of people willing to work under the new regime is equal to the (fixed) number of jobs:

$$\sum_i \pi_i(R, v_R, \mu_0(v_R)) = J(v_T) \quad (19)$$

The implementation of the equilibrium procedure requires to specify how  $J$  and  $w_i$  depend on  $v$ . In principle, given appropriate identification conditions (for example with panel data) and a suitable empirical specification for  $J(v)$ , it might be possible to estimate it by substituting  $\mu_0 = \ln\left(\frac{J(v)}{\gamma g_0}\right)$  into the choice probabilities (12). In the empirical example of Section 5, for illustrative purposes we will adopt the simple assumption that  $J$  depends on the

mean of the wage distribution according to a constant-elasticity relationship such as  $J = K\omega^{-\eta}$ , where  $\omega$  is the mean of the wage rate distribution,  $-\eta$  is the elasticity of labour demand and  $K$  is a constant. Individual wage rates are shifted together with the mean  $\omega$  and maintain the same rank position in the distribution. We will perform a sort of sensitivity-analysis by imputing alternative values to the elasticity of labour demand.

#### 4. Extensions

The basic framework illustrated above can be extended in many directions.

##### 4.1. *Non uniform density of market jobs*

As in expression (8), we might want to specify a non-uniform conditional density for the market jobs. Let us consider again a single person. In this case we write  $J = J(\vartheta)$  and  $J_\ell = J_\ell(\vartheta)$ ,  $\ell = 1, \dots, L$ , which implies the relationships  $\mu_0 = \mu_0(\vartheta)$  and  $\mu_\ell = \mu_\ell(\vartheta)$ .

We then define the probability that individual  $i$  is matched to a market job of type  $j \in S_\ell$  as

$$\begin{aligned} \pi_i^\ell(T, \vartheta_T, \mu_0(\vartheta_T), \mu_2(\vartheta_T), \dots, \mu_L(\vartheta_T)) &\equiv \\ &\equiv \sum_{j \in S_\ell} \frac{\exp\{V(i, j; w_i(\vartheta_T) + \mu_0(\vartheta_T) + \mu_\ell(\vartheta_T))\}}{\sum_{k=0}^H \exp\{V(i, k; w_i(\vartheta_T), T) + \mu_0(\vartheta_T)D(k > 1) + \sum_{l=2}^L \mu_l(\vartheta_T)D(k \in S_l)\}} \end{aligned} \quad (20)$$

with  $\mu_1 = 0$ . The probability that individual  $i$  is matched to a market job is

$$\pi_i(T, \vartheta_T, \mu_0(\vartheta_T), \mu_2(\vartheta_T), \dots, \mu_L(\vartheta_T)) \equiv \sum_{\ell=1}^L \pi_i^\ell(T, \vartheta_T, \mu_0(\vartheta_T), \mu_2(\vartheta_T), \dots, \mu_L(\vartheta_T)). \quad (21)$$

The equilibrium conditions for a reform  $R$  are respectively:

$$\begin{aligned} \sum_i \pi_i(R, \vartheta_R, \mu_0(\vartheta_R), \mu_2(\vartheta_R), \dots, \mu_L(\vartheta_R)) &= \gamma g_0 e^{\mu_0(\vartheta_R)} \\ \sum_i \pi_i^\ell(R, \vartheta_R, \mu_0(\vartheta_R), \mu_2(\vartheta_R), \dots, \mu_L(\vartheta_R)) &= \gamma^2 g_0 n(S_\ell) e^{\mu_0(\vartheta_R) + \mu_\ell(\vartheta_R)}, \ell = 2, \dots, L \end{aligned} \quad (22)$$

with elastic demand;

$$\begin{aligned}\sum_i \pi_i(R, \vartheta_T, \mu_{0R}, \mu_{2R}, \dots, \mu_{LR}) &= \gamma g_0 e^{\mu_{0R}} \\ \sum_i \pi_i^\ell(R, \vartheta_T, \mu_{0R}, \mu_{2R}, \dots, \mu_{LR}) &= \gamma^2 g_0 n(S_\ell) e^{\mu_{0R} + \mu_{\ell R}}, \ell = 2, \dots, L.\end{aligned}\quad (23)$$

with perfectly elastic demand and

$$\begin{aligned}\sum_i \pi_i(R, \vartheta_R, \mu_0(\vartheta_R), \mu_2(\vartheta_R), \dots, \mu_L(\vartheta_R)) &= \gamma g_0 e^{\mu_0(\vartheta_R)} \\ \sum_i \pi_i^\ell(R, \vartheta_R, \mu_0(\vartheta_R), \mu_2(\vartheta_R), \dots, \mu_L(\vartheta_R)) &= \gamma^2 g_0 n(S_\ell) e^{\mu_0(\vartheta_R) + \mu_\ell(\vartheta_R)}, \ell = 2, \dots, L.\end{aligned}\quad (24)$$

with perfectly inelastic demand.

#### 4.2. Couples

When analyzing the simultaneous labour supply decisions of married couples we might want to distinguish the choice set available to males (M) and females (F). The previous notation and the choice probabilities are generalized accordingly:

$$P(i, j_F, j_M; T) = \frac{\exp \left\{ V(i, j_F, j_M; w_{iF}, w_{iM}, T) + \sum_{x=F, M} \left( \mu_{0x} D(j_x > 0) + \sum_{\ell=2}^L \mu_{\ell x} D(j_x \in S_{\ell x}) \right) \right\}}{\sum_{k=0}^H \exp \left\{ V(i, k_F, k_M; w_{iF}, w_{iM}, T) + \sum_{x=F, M} \left( \mu_{0x} D(k_x > 0) + \sum_{\ell=2}^L \mu_{\ell x} D(k_x \in S_{\ell x}) \right) \right\}}. \quad (25)$$

For  $x = F$  or  $M$ , expression (10) is generalized as follows:

$$\mu_{0x} = \ln \left( \frac{J_x}{\gamma g_{0x}} \right), \mu_{\ell x} = \ln \left( \frac{J_{\ell x} / J_x}{\gamma n(S_{\ell x})} \right), \ell = 2, \dots, L_x. \quad (26)$$

We then specify gender-specific labour demand functions:

$$J_x = J_x(\vartheta) \quad (27)$$

$$J_{\ell x} = J_{\ell x}(\vartheta), \ell = 2, \dots, L_x \quad (28)$$

where  $\vartheta$  now denotes the moments of the joint distribution of the partners' wage rates.

Expressions (26), (27) and (28) imply mappings such as:

$$\mu_{0x} = \mu_{0x}(\vartheta), \mu_{\ell x} = \mu_{\ell x}(\vartheta), \ell = 2, \dots, L_x. \quad (29)$$

Let us define  $\pi_{ix}^\ell(T, \vartheta_T, \mu_{0F}(\vartheta_T), \mu_{0M}(\vartheta_T), \mu_{2F}(\vartheta_T), \dots, \mu_{LF}(\vartheta_T), \mu_{2M}(\vartheta_T), \dots, \mu_{LM}(\vartheta_T))$  as the probability that the partner of gender  $x$  in couple  $i$  is matched to a job  $j_x \in S_{\ell x}$ , given the (current) tax-transfer regime T. Then

$$\begin{aligned} \pi_{ix}^\ell(T, \vartheta_T, \mu_{0F}(\vartheta_T), \mu_{0M}(\vartheta_T), \mu_{2F}(\vartheta_T), \dots, \mu_{LF}(\vartheta_T), \mu_{2M}(\vartheta_T), \dots, \mu_{LM}(\vartheta_T)) &= \\ = \sum_{\ell=1}^{L_x} \pi_{ix}^\ell(T, \vartheta_T, \mu_{0F}(\vartheta_T), \mu_{0M}(\vartheta_T), \mu_{2F}(\vartheta_T), \dots, \mu_{LF}(\vartheta_T), \mu_{2M}(\vartheta_T), \dots, \mu_{LM}(\vartheta_T)) & \end{aligned} \quad (30)$$

is the probability that partner of gender  $x$  in couple  $i$  is matched to a market job.

Then the equilibrium conditions are

$$\begin{aligned} \sum_i \pi_{ix}(T, \vartheta_R, \mu_{0F}(\vartheta_R), \mu_{0M}(\vartheta_R), \mu_{2F}(\vartheta_R), \dots, \mu_{LF}(\vartheta_R), \mu_{2M}(\vartheta_R), \dots, \mu_{LM}(\vartheta_R)) &= \gamma g_{0x} e^{\mu_{0x}(\vartheta_R)}, \\ \sum_i \pi_{ix}^\ell(T, \vartheta_R, \mu_{0F}(\vartheta_R), \mu_{0M}(\vartheta_R), \mu_{2F}(\vartheta_R), \dots, \mu_{LF}(\vartheta_R), \mu_{2M}(\vartheta_R), \dots, \mu_{LM}(\vartheta_R)) &= \gamma^2 g_{0x} n(S_{\ell x}) e^{\mu_{0x}(\vartheta_R) + \mu_{\ell x}(\vartheta_R)}, \\ x = F, M, \ell = 2, \dots, L & \end{aligned} \quad (31)$$

for the case with elasticity demand;

$$\begin{aligned} \sum_i \pi_{ix}(T, \vartheta_R, \mu_{0FR}, \mu_{0MR}, \mu_{2FR}, \dots, \mu_{LFR}, \mu_{2MR}, \dots, \mu_{LMR}) &= \gamma g_{0x} e^{\mu_{0xR}}, \\ \sum_i \pi_{ix}^\ell(T, \vartheta_R, \mu_{0FR}, \mu_{0MR}, \mu_{2FR}, \dots, \mu_{LFR}, \mu_{2MR}, \dots, \mu_{LMR}) &= \gamma^2 g_{0x} n(S_{\ell x}) e^{\mu_{0xR} + \mu_{\ell xR}}, \\ x = F, M, \ell = 2, \dots, L & \end{aligned} \quad (32)$$

for the case with perfectly elastic demand and

$$\begin{aligned} \sum_i \pi_{ix}(T, \vartheta_R, \mu_{0F}(\vartheta_R), \mu_{0M}(\vartheta_R), \mu_{2F}(\vartheta_R), \dots, \mu_{LF}(\vartheta_R), \mu_{2M}(\vartheta_R), \dots, \mu_{LM}(\vartheta_R)) &= \gamma g_{0x} e^{\mu_{0x}(\vartheta_T)}, \\ \sum_i \pi_{ix}^\ell(T, \vartheta_R, \mu_{0F}(\vartheta_R), \mu_{0M}(\vartheta_R), \mu_{2F}(\vartheta_R), \dots, \mu_{LF}(\vartheta_R), \mu_{2M}(\vartheta_R), \dots, \mu_{LM}(\vartheta_R)) &= \gamma^2 g_{0x} n(S_{\ell x}) e^{\mu_{0x}(\vartheta_T) + \mu_{\ell x}(\vartheta_T)}, \\ x = F, M, \ell = 2, \dots, L & \end{aligned} \quad (33)$$

for the case with perfectly inelastic demand.

### 4.3 Matching equilibrium

The matching model developed by Dagsvik (2000) replaces the simple concept of equilibrium adopted in this note with the notion of stable matching. Our equilibrium is a special case of a

stable matching where the number of realized matches is equal to the number of available jobs and to the number of people willing to work. More generally, however, we can have a stable matching that involves vacancies and unemployment.

#### ***4.4 Changes in non-market opportunities***

So far we have treated  $g_0$  (defined in Section 2) as a constant. It might be argued that when reaching a new equilibrium, also  $g_0$  might change. For example it might be the case that market jobs provide also goods and services that are complements or substitutes to non-market activities: thus changes in the number of market jobs might induce a change in  $g_0$ . If we make the (very special) assumptions that  $g_0$  varies in the same proportion as  $J$  and if labour demand is perfectly elastic, then we have a scenario where both the wage rate and  $\mu_0$  remain constant, thus providing an equilibrium interpretation of the standard simulation procedure.

## 5. An empirical illustration

We illustrate the procedure presented above with a simulation of various hypothetical reforms of income support in Italy, using a microeconomic model of household labour supply. The model, the estimates, the policy motivations and the simulated reforms are fully described in Colombino (2011). Here we illustrate the main features of the model and some of the simulation results with the perspective of illustrating the implications of the equilibrium simulation.

### 5.1. The model

We consider households with two decision-makers (couples) or one decision-maker (singles).

The choices of other people – if any – in the household are taken as exogenous.

The choice probabilities for singles and couples are those of expressions (20) and (25) respectively.

Each individual (single or partner in a couple) chooses among 11 job-types defined by weekly hours of work  $h$ : so  $h_0 = 0$  and  $h_1, h_2, \dots, h_{10}$  are ten random values drawn from the intervals 1-8, 9-16, 17-24, 25-32, 33-40, 41-48, 49-56, 57-64, 65-72, 73-80.

For the systematic part of the utility function we adopt a quadratic specification, where  $C$  denotes household total net available income:

$$\begin{aligned} V = & \theta_C C + \theta_F (T - h_F) + \theta_M (T - h_M) + \theta_{CC} C^2 + \theta_{FF} (T - h_F)^2 + \\ & + \theta_{MM} (T - h_M)^2 + \theta_{CF} C(T - h_M) + \theta_{CM} C(T - h_M) + \theta_{FM} (T - h_F)(T - h_M) \end{aligned} \quad (34)$$

for couples and

$$V = \theta_C C + \theta_x (T - h_x) + \theta_{CC} C^2 + \theta_{xx} (T - h_x)^2 + \theta_{Cx} C(T - h_x) \quad (35)$$

for singles ( $x = F, M$ ).

Some of the above parameters  $\theta$  are made dependent on characteristics:

$$\begin{aligned}
\theta_F &= \beta_{F0} + \beta_{F1} (\text{Age of the wife}) + \beta_{F2} (\text{Age of the wife})^2 + \\
&\quad + \beta_{F3} (\# \text{Children}) + \beta_{F4} (\# \text{Children under 6}) + \beta_{F5} (\# \text{Children 6-10}) \\
\theta_M &= \beta_{M0} + \beta_{M1} (\text{Age of the husband}) + \beta_{M2} (\text{Age of the husband})^2 + \\
&\quad + \beta_{M3} (\# \text{Children}) + \beta_{M4} (\# \text{Children under 6}) + \beta_{M5} (\# \text{Children 6-10}) \\
\theta_C &= \beta_{C0} + \beta_{C1} (\text{Household's size}) \\
\theta_x &= \beta_{x0} + \beta_{x1} (\text{Age}) + \beta_{x2} (\text{Age})^2 + \beta_{x3} (\# \text{Children}) + \\
&\quad + \beta_{x4} (\# \text{Children under 6}) + \beta_{x5} (\# \text{Children 6-10}), x = F, M.
\end{aligned} \tag{36}$$

In order to compute the value of  $C$  for all the job-types we use the EUROMOD

Microsimulation model. Wage rates for those who are observed as not employed are imputed on the basis of a wage equation estimated on the employed subsample and corrected for sample selection.

For the estimation and simulation exercise we use a EUROMOD dataset produced from the 1998 Survey of Household Income and Wealth (SHIW1998).<sup>2</sup>

The data include couples and singles. Both partners of couple households and heads of single households are aged 20 – 55 and are wage employed, self-employed, unemployed or inactive (students and disabled are excluded). As a result we are left with 2955 couples, 366 single females and 291 single males.

The simulation exercise accounts for equilibrium between the total number of jobs and the number of people willing to work. The implicit (simplifying) assumption in the exercise is that, whilst the number of jobs and people willing to work are equated by the equilibrium wage distribution, the hours worked accommodate households' preferences. For gender  $x = F$ ,  $M$  we adopt the following empirical specification for expression (27):

$$J_x = K_x \omega_x^{-\eta} \tag{37}$$

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<sup>2</sup> More recent datasets are of course available. We chose to use a model that was already estimated on 1998 data with the purpose of illustrating a methodological proposal. From the perspective of the policy simulations, pre-2001 data do not suffer from the turbulent macroeconomic scenarios that characterize the post-2001 years.



where  $\omega_x$  is the mean of the wage rates distribution,  $K_s$  is a constant and  $-\eta$  is the elasticity of labour demand. Therefore:

$$\mu_{0x}(\omega_x) = \ln \left( \frac{K_x \omega_x^{-\eta}}{\gamma g_{0x}} \right) \quad (38)$$

Given  $J_x$  (observed or simulated under the current tax-transfer system),  $\omega_x$ , the estimated  $\mu_{0x}$  and an imputed value of  $\eta$ , we can use expressions (37) and (38) to retrieve  $\gamma g_{0x}$  and  $K_x$ .

In this exercise we use  $\eta = 0, 0.5, 1, \infty$ .

The equilibrium conditions derived in Section 4 are fulfilled by calibrating  $\omega_x$  (i.e. shifting the location of the wage rate distributions) or directly  $\mu_{0x}$  (when  $\eta = \infty$ ) in the course of the simulation.

## 5.2. The policies

Current Italian income support policies can be classified as contingent interventions (such as unemployment benefits) and structural (or anti-poverty) interventions.

There appear to be three main undesirable features of the design of contingent policies: (a) being they aimed at preserving the job rather than the worker's income and opportunities, the labour reallocation from unprofitable jobs to more promising ones is severely discouraged; (b) they are limited to certain sector and types of contract, thus generating social exclusion and processes of the insider-outsider type; (c) often some of the contingent interventions have to go through a bargaining process involving firms, unions and local or central authorities, thus adding more sources of potential inequities.

The anti-poverty interventions are mainly aimed at supporting low pensions, disabled people and low-income families with a mean-tested transfer, which is however limited to wage employees. Embodied in the personal income taxation system there are also tax credits

and child benefits that can be classified as anti-poverty policies. It has been observed that the design of these policies creates distortions and bad incentives for labour market participations of married women (e.g. Colonna and Marcassa, 2011).

Overall, many analysts have suggested that the current Italian system of income support policies is defective with respect to both efficiency goals (e.g. minimizing distortions and supporting labour mobility) and equity goals (e.g. reducing poverty and economic insecurity).<sup>3</sup>

In this paper we consider various versions of hypothetical income support policies that – differently from the current policies described above – are universal, meaning that they are not conditional upon professional or occupational categories or on bargaining or contingent financial constraints. As it is typically the case with universal policies, they are financed by general taxes. These reforms are stylized cases representative of the different scenarios that are discussed or even actually implemented in many countries.

In the following description of the policies there appears a “threshold”  $G$  that will be defined below.

**Guaranteed Minimum Income (GMI).** Each individual receives a transfer equal to  $G - I$  if single or  $G/2 - I$  if partner in a couple provided  $I < G$  (or  $I < G/2$ ), where  $I$  denotes individual taxable income. This is the standard conditional (or means-tested) income support mechanism.

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<sup>3</sup> See for example Onofri (1997), Baldini et al. (2002), Boeri and Perotti (2002) and Sacchi (2005). A first microeconomic evaluation of alternative reforms of the Italian tax-transfer system was done by Aaberge et al. 2004). In March 2012 the Italian Government has designed a reform of the income support policies, which at the moment is being discussed by the Parliament. The reform contains some steps toward universalism but so far it does not seem to change the basic characteristics of the current system.

**Unconditional Basic Income (UBI).** Each individual receives an unconditional transfer equal to  $G$  if single or  $G/2$  if partner in a couple. It is the basic version of the system discussed for example by Van Parijs (1995) and also known in the policy debate as “citizen income” or “social dividend” (Meade 1995; Van Trier 1995).

**Wage Subsidy (WS).** Each individual receives a 10% subsidy on the gross hourly wage and her/his income is not taxed as long as her/his gross income (including the subsidy) does not exceed  $G$  if single or  $G/2$  if partner in a couple. This is close to various in-work benefits or tax-credits reforms introduced for example in the USA (Earned Income Tax Credit), in the UK (In-Work Benefits) and in Sweden.<sup>4</sup>

**GMI + WS** and **UBI + WS** are mixed mechanisms where the transfer is coupled with the wage subsidy, but with the threshold redefined as  $0.5G$ .<sup>5</sup>

In order to define  $G$ , let us preliminary define

$C$  = total net available income (current) of the household:

$N$  = total number of components of household  $n$ .

$\tilde{C} = C/\sqrt{N}$  = “individual-equivalent” income.

$P = \text{median}(\tilde{C})/2$  = Poverty Line.

Then:

$$G = aP\sqrt{N},^6$$

where  $a \in [0,1]$  is a “coverage” rate, i.e. what proportion of the poverty line is covered by  $G$ .

For each reform we simulate three versions with different values of  $a$ : 1, 0.75 and 0.50. For

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<sup>4</sup> Many authors have recently analysed or suggested in-work-benefits policies for Italy (Colonna and Marcassa 2011, Figari 2011, De Luca et al. 2012)

<sup>5</sup> A mixed system close to GMI+WS has been proposed in Italy by De Vincenti and Paladini (2009).

<sup>6</sup> The “square root scale” is one of the equivalence scales commonly used in OECD publications.

example,  $G=0.5P\sqrt{3}$  means that for a household with 3 components the threshold is  $\frac{1}{2}$  of the Poverty Line times the equivalence scale  $\sqrt{3}$ .

The income support mechanism is matched with a progressive tax that replicates the current system but with marginal tax rates applied to the whole income exceeding  $G$  (or  $G/2$ ) and proportionally adjusted according to a constant  $\tau$  (the parameter  $\tau$  is used in the simulation as a calibrating device in order to fulfil the public budget constraint).

Altogether we have  $5$  (types)  $\times 3$  (values of  $a$ ) =  $15$  reforms.

Each reform defines a new budget constraint for each household. The simulation consists of running the model after replacing the current budget constraint with the reformed one. The parameter  $\tau$  (defined above) is endogenously determined so that the total net tax revenue is equal to the one collected under the current tax-transfer system (taking into account the households' behavioural responses). The equilibrium conditions are attained by iteratively calibrating the mean of the wage rate distribution: this will determine the number of available market jobs through expression (37) and the value of  $\mu_0$  (expression (38)), which in turn will affect the number of people accepting a job (expression (38)). Besides the 15 alternative reforms we also simulate a tax-transfer system – that we call “current” – with the same five alternative procedures used for the reforms: it is characterized by the same income support mechanism as in the true current system, but the tax rule is given a simplified representation as in the reforms: namely, we apply the marginal tax rates to the whole personal income. Therefore we compare what would happen with this system and with the reforms under the alternative equilibrium conditions. We think this procedure is preferable to the standard one consisting of comparing the observed *status quo* to the reforms.<sup>7</sup>

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<sup>7</sup> The results reported in Colombino (2011) are in part different from the ones reported here since the current system is defined there as the observed *status quo*.

Five simulation procedures are adopted: one where the equilibrium conditions are ignored and four more where the equilibrium conditions are determined by

$$\eta = 0, -0.5, -1.0, \infty.$$

We evaluate the policies with a Social Welfare function defined as:

$$(\text{Average Individual Welfare}) \times (1 - \text{Gini index of the distribution of Individual Welfare}).$$

This is similar to the so-called Sen Social Welfare index and it can be rationalized as a member of a rank-dependent social welfare indexes (e.g. Aaberge and Colombino 2012a, 2012b). Individual Welfare is the money metric equivalent of the expected maximum utility (EMU). The EMU is the natural logarithm of the denominator of the choice probabilities. The money metric equivalent for household  $i$  is the level of income that makes the EMU of the reference household (we choose the worst-off one) equal to the EMU of household  $i$  (King 1983).

### 5.3. Results

Tables 1 – 5 report some results of the five simulations

The policies are ranked in descending order (best one at the top) according to the Social Welfare function defined in Section 5.2. The reforms are identified by the content of the first two columns: the income support mechanism (GMI etc.) and the coverage, i.e. the value of  $a$  (0.5, 0.75 or 1) defined in section 5.2. For example, (UBI+WS, 0.75) denotes a policy where the income support mechanism is UBI+WS and  $G$  is 75% of the Poverty line.

For each reform we report three pieces of information related to behavioural effects (annual hours of work), distortions (top marginal tax rate) and distributive effects (poverty rate).

The different simulation procedures lead to notable differences in the results. The standard (no equilibrium) procedure seems to favour a more generous coverage: out of the five best policies of Table 1, two have  $a = 1$ , two have  $a = 0.75$  and one has  $a = 0.5$ . In the other Tables the average coverage among the first five best policies is lower and it decreases with respect to  $\eta$ . The no equilibrium procedure favours also pure unconditional policies: three positions out of the first five of Table 1 are occupied by UBI policies. On the contrary, when we assume  $\eta = 0$ , three out of the first five policies are mean-tested (GMI). In the other cases, the results are more mixed, with some prevalence of UBI+WS policies. The current mechanism of income support is always ranked at the bottom, except when  $\eta = \infty$ . With increasing  $\eta$ , less generous policies – including the current one – move up in the ranking. This happens because a more elastic labour demand moderates the increase in equilibrium wages, which in turn implies higher equilibrium tax rates. In most cases the income effects induced by the reforms appear to work in opposite directions for females and males: the reforms induce more (less) hours worked by of women (men) when compared to the current system, the exception being again the simulation with  $\eta = \infty$ , where, under the three worst policies, women work fewer hours than under the current system.

We have noted in Section 3 that the common practice of not accounting for the equilibrium adjustment of the wage rates is usually interpreted as a perfectly elastic demand scenario. This interpretation is not correct: indeed by comparing Table 1 and Table 5 we see that the simulation performed under the correctly specified scenario with perfectly elastic demand produces results that are radically different from those produced by the no-equilibrium simulation.

## **6. Conclusions**

The standard simulation procedure adopted when using microeconomic models of labour supply for reform evaluation might not be consistent with an interpretation of the simulation results in terms of comparative statics, i.e. comparison of different equilibria. This happens when the model includes a representation of aspects of the pre-reform equilibrium (such as the availability of different types of jobs) that are going to change in the post-reform equilibrium and when this change is not properly accounted for. We have proposed a simulation method that takes into account such a change for a certain class of microeconomic models and leads to a consistent interpretation of the simulation results as an exercise in comparative statics. We have illustrated the relevance of the different simulation procedures with an evaluation of alternative reforms of the Italian income support policies.

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The empirical example of this paper uses EUROMOD (Ver. 2.7a). EUROMOD is a tax-benefit microsimulation model for the European Union that enables researchers and policy analysts to calculate, in a comparable manner, the effects of taxes and benefits on household incomes and work incentives for the population of each country and for the EU as a whole. EUROMOD was originally designed by a research team under the direction of Holly Sutherland at the Department of Economics in Cambridge, UK. It is now developed and updated at the Microsimulation Unit at ISER (University of Essex, UK).

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<b>Table 1. No equilibrium</b>					
<i>Income Support Mechanism</i>	<i>Coverage</i>	<b>Annual Average Hours of Work (Women)</b>	<b>Annual Average Hours of Work (Men)</b>	<b>Top Marginal Tax Rate (%)</b>	<b>Head Count Poverty Ratio</b>
<b>UBI+WS</b>	1.00	999	2042	53.6	0.17
<b>UBI</b>	0.75	991	2039	55.4	0.04
<b>UBI+WS</b>	0.75	1004	2043	51.3	1.01
<b>UBI</b>	0.50	1000	2042	50.9	0.52
<b>UBI</b>	1.00	982	2036	59.9	0.00
<b>WS</b>	1.00	1016	2046	48.2	3.38
<b>UBI+WS</b>	0.50	1008	2045	49.5	2.44
<b>WS</b>	0.75	1015	2046	47.3	3.67
<b>WS</b>	0.50	1016	2047	46.8	4.11
<b>GMI+WS</b>	1.00	1000	2043	50.9	1.43
<b>GMI+WS</b>	0.75	1005	2044	49.1	2.34
<b>GMI+WS</b>	0.50	1008	2045	47.7	3.26
<b>GMI</b>	0.50	1000	2044	45.9	2.36
<b>GMI</b>	1.00	983	2039	51.3	0.01
<b>GMI</b>	0.75	992	2042	48.2	0.87
<b>Current</b>		945	2063	43.7	4.33

Table 2. Equilibrium with $\eta = 0$ .					
<i>Income Support Mechanism</i>	<i>Coverage</i>	<b>Annual Average Hours of Work (Women)</b>	<b>Annual Average Hours of Work (Men)</b>	<b>Top Marginal Tax Rate (%)</b>	<b>Head Count Poverty Ratio</b>
<b>GMI</b>	1	1005	2046	48.2	0.26
<b>GMI</b>	0.75	1009	2047	45.7	1.32
<b>UBI</b>	0.5	1009	2044	50.0	0.62
<b>GMI</b>	0.5	1013	2048	44.9	2.95
<b>UBI+WS</b>	0.5	1014	2048	48.9	2.64
<b>UBI+WS</b>	0.75	1010	2046	49.9	1.00
<b>UBI+WS</b>	1	1005	2044	52.3	0.24
<b>GMI+WS</b>	1	1010	2046	49.7	1.03
<b>UBI</b>	0.75	996	2040	54.7	0.06
<b>GMI+WS</b>	0.75	1010	2046	48.0	2.42
<b>GMI+WS</b>	0.5	1014	2047	47.4	3.41
<b>WS</b>	0.5	1016	2047	46.8	4.15
<b>WS</b>	1	1017	2047	48.0	2.99
<b>WS</b>	0.75	1014	2047	46.9	3.63
<b>UBI</b>	1	971	2032	62.0	0.00
<b>Current</b>		945	2063	43.7	4.33

Table 3. Equilibrium with $\eta = 0.5$ .					
<i>Income Support Mechanism</i>	<i>Coverage</i>	Annual Average Hours of Work (Women)	Annual Average Hours of Work (Men)	Top Marginal Tax Rate (%)	Head Count Poverty Ratio
<b>UBI+WS</b>	0.75	1009	2044	50.2	0.95
<b>UBI+WS</b>	0.5	1013	2046	49.2	2.52
<b>WS</b>	0.75	1020	2047	46.5	3.65
<b>WS</b>	0.5	1019	2047	46.6	4.14
<b>GMI+WS</b>	0.5	1012	2046	47.7	3.40
<b>GMI+WS</b>	0.75	1009	2045	48.3	2.38
<b>WS</b>	1	1019	2046	47.9	3.04
<b>UBI</b>	0.5	1002	2042	50.9	0.52
<b>UBI</b>	0.75	993	2038	55.3	0.04
<b>UBI+WS</b>	1	1004	2040	53.0	0.18
<b>UBI</b>	1	984	2035	59.8	0.00
<b>GMI</b>	0.5	1005	2045	45.8	2.48
<b>GMI</b>	0.75	996	2043	47.3	0.81
<b>GMI+WS</b>	1	1004	2043	50.7	0.79
<b>GMI</b>	1	986	2040	50.9	0.00
<b>Current</b>		945	2063	43.7	4.33

Table 4. Equilibrium with $\eta = 1.0$ .					
<i>Income Support Mechanism</i>	<i>Coverage</i>	<b>Annual Average Hours of Work (Women)</b>	<b>Annual Average Hours of Work (Men)</b>	<b>Top Marginal Tax Rate (%)</b>	<b>Head Count Poverty Ratio</b>
<b>UBI+WS</b>	0.75	1011	2043	50.2	0.95
<b>UBI+WS</b>	0.5	1014	2045	49.2	2.52
<b>WS</b>	0.75	1021	2046	46.5	3.66
<b>WS</b>	0.5	1021	2047	46.6	4.14
<b>GMI+WS</b>	0.5	1013	2046	47.6	3.40
<b>GMI+WS</b>	0.75	1010	2045	48.3	2.38
<b>WS</b>	1	1020	2046	47.9	3.04
<b>UBI</b>	0.5	1003	2041	50.8	0.52
<b>UBI</b>	0.75	994	2038	55.2	0.04
<b>UBI+WS</b>	1	1005	2040	52.9	0.20
<b>UBI</b>	1	985	2034	59.7	0.00
<b>GMI</b>	0.5	1005	2044	45.7	2.48
<b>GMI+WS</b>	1	1005	2043	50.6	0.79
<b>GMI</b>	0.75	997	2042	47.2	0.81
<b>GMI</b>	1	988	2039	51.1	0.01
<b>Current</b>		945	2063	43.7	4.33

<b>Table 5. Equilibrium with <math>\eta = \infty</math>.</b>					
<i>Income Support Mechanism</i>	<i>Coverage</i>	<b>Annual Average Hours of Work (Women)</b>	<b>Annual Average Hours of Work (Men)</b>	<b>Top Marginal Tax Rate (%)</b>	<b>Head Count Poverty Ratio</b>
<b>UBI+WS</b>	0.5	982	2044	49.3	2.53
<b>GMI+WS</b>	0.5	981	2045	47.7	3.31
<b>WS</b>	0.5	985	2046	46.6	4.16
<b>UBI</b>	0.5	969	2040	50.9	0.46
<b>GMI</b>	0.5	971	2043	45.9	2.44
<b>Current</b>		914	2062	43.7	4.42
<b>UBI+WS</b>	0.75	948	2040	50.4	0.85
<b>GMI+WS</b>	0.75	947	2042	48.6	2.19
<b>WS</b>	0.75	953	2044	46.6	3.64
<b>UBI</b>	0.75	928	2035	55.5	0.02
<b>GMI</b>	0.75	931	2039	47.7	0.72
<b>UBI+WS</b>	1	912	2036	53.3	0.05
<b>WS</b>	1	922	2041	48.1	2.95
<b>GMI+WS</b>	1	911	2038	51.2	0.73
<b>UBI</b>	1	888	2030	60.3	0.00
<b>GMI</b>	1	890	2034	52.1	0.00